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The evolution of Patinkin's interpretation of Keynes' principle of effective demand

Jochen Hartwig

ABSTRACT

Don Patinkin (1922–1995) was both an eminent theoretical economist and a great historian of economic thought. In the latter field, his focus was on Keynes' "principle of effective demand" from Chapter 3 of the *General Theory*. Having submitted a first interpretation of the "principle" in 1976 – in which he claimed that it contains major flaws – Patinkin revisited the subject several times over the next couple of years. In this process, his interpretation changed markedly. The aim of this paper is to trace (and to comment on) the evolution of Patinkin's interpretation of the theory of effective demand.

KEYWORDS

Patinkin; Keynes; principle of effective demand; *D/Z* model

JEL CLASSIFICATION

B22; B31; E12

1. Introduction

"Keynes' Chapter 3 on 'The Principle of Effective Demand'", Patinkin (1979a, 155) writes, "is at one and the same time the most important and the most obscure chapter in the *General Theory*: most important, because it contains the major innovation of the book … ". And obscure, indeed, it is also – as is evidenced by the incessant debate on the "correct" interpretation of the model of effective demand Keynes presents in this chapter – the *D/Z* model – to which I have contributed in a dozen publications over the past 20 years.[1] That this debate soldiers on bears evidence to the utmost importance – as pointed out by Patinkin – of understanding the "major innovation" of the *General Theory*.

Revisiting Patinkin's interpretation seems rewarding, not only because he is "widely regarded as one of the leading experts" on the Keynesian revolution (Backhouse 2002, 186), but also because his interpretation of Keynes' principle of effective demand changed substantially over time. Having submitted a first interpretation of this model in 1976 – in which he claimed that it contains major flaws – Patinkin revisited the *D/Z*

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[1] King (1994) scrutinizes the early stages of this debate. "To conclude that there was some confusion about aggregate supply and demand analysis in the early 1950s would be a grotesque understatement", he sums up (14).

model several times over the next couple of years. In this process, his interpretation changed markedly. The objective of this paper is to describe and to comment on the evolution of Patinkin's interpretation of Keynes' principle of effective demand over a period of time that covered more than thirty years of his academic career.

In reconstructing the evolution of Patinkin's interpretation of Keynes' principle of effective demand, the paper covers a niche in the literature on the economics of Patinkin that – unlike many other aspects – has not come under much scrutiny yet. Earlier literature has compared Patinkin's views with those of his contemporaries Haavelmo, Klein, Lange, Marschak, and Modigliani (Boianovsky 2002; Rubin 2002, 2004; Hagemann 2017).[2] Patinkin's impact on the development of disequilibrium macroeconomics in the 1970s is also well-researched (Rubin 2012; Backhouse and Boianovsky 2013). The only other contribution that deals with Patinkin's interpretation of Chapter 3 of Keynes' *General Theory* is Arthmar and Brady (2009). These authors, however, instead of systematically reconstructing the evolution of Patinkin's interpretation, rather "briefly review certain points raised by Patinkin" (Arthmar and Brady 2009, 128). In particular, they confine themselves to discussing Patinkin's (changing) interpretation of the aggregate supply function Z, remaining largely silent about his views on the aggregate demand function D, which were also in flux.[3]

The paper is structured chronologically. The next section discusses Patinkin's early engagement with the matter, that is, before the publication of volumes XIII and XIV of the *Collected Writings of John Maynard Keynes*. It will be seen that these early contributions do not address Keynes' Chapter 3 model yet. Section III scrutinises *Keynes' Monetary Thought* (KMT) and two subsequent articles that appeared in the journal *History of Political Economy* in 1977 and 1978 in which Patinkin published a correction of an error in and a reply to a reviewer of the 1976 book, respectively. Taken together, these three writings present a coherent interpretation of Keynes' theory of effective demand that Patinkin moved away from, however, in his 1979 *Economic Inquiry* article already mentioned above. In Patinkin (1982a) he consolidated his new interpretation. Section IV examines this reorientation. Section V concludes.

2. Early views

In Chapter 3 of the *General Theory*, Keynes introduces two functions, the *aggregate supply function* (Z) and the *aggregate demand function* (D), both being functions of the level of employment (N). So the title of Patinkin's (1949) article "Involuntary unemployment and the Keynesian supply function" seems to indicate that he wanted to make inroads into the theory of effective demand set out in this chapter. However, Chapter 3 is not Patinkin's point of reference in his 1949 article. He rather suggests to

[2] Rancan (2017, 162, fn. 28) considers especially Lange's and Marschak's influence on Patinkin as "already reconstructed".

[3] Also note that Arthmar and Brady's main intention it is to prove that the slope of Z can be derived mathematically from Chapter 20 of Keynes' *General Theory*. Their section on Patinkin, which serves as a "preamble" (Arthmar and Brady 2009, 127) to that proof, sets the stage for Arthmar and Brady's rebuttal of Patinkin's claim as to "Keynes' supposed lack of mathematical proficiency" (Arthmar and Brady 2009, 129).

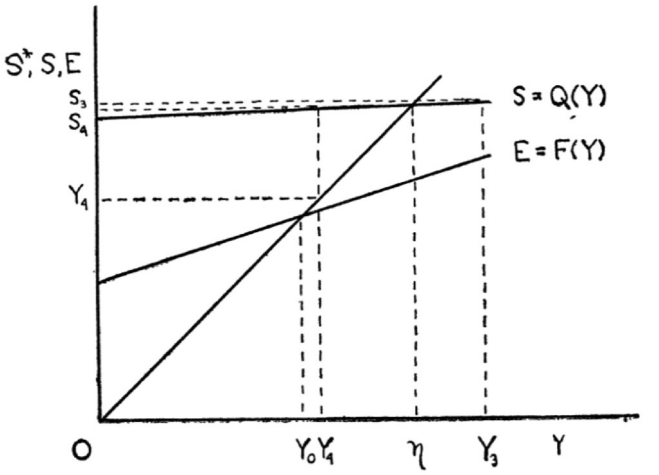

Figure 1. Aggregate supply curve in the Keynesian cross I (source: Patinkin 1949, Figure 3).

augment the familiar "Keynesian cross" – which he calls "standard Keynesian analysis" (Patinkin 1949, 364) without making any reference to Samuelson (1948) who passes for the originator of that model[4] – with an aggregate supply function built up from "Walrasian supply functions" in individual markets (Patinkin 1949, 365). Patinkin calls it the "aggregate desired-supply function" (Patinkin 1949, 365) and suggests that this function $S = Q(Y)$ might be a horizontal line in the Keynesian cross diagram.[5] However, he prefers to give it a small positive slope with respect to real income/output (Y). At any rate, the aggregate supply function normally (except in wartime that means) lies above the expenditure function $E = F(Y)$ (being the vertical sum of the consumption and the investment function) so that it intersects the 45°-line at a higher level of output than the expenditure function (Figure 1).

The point of intersection of the aggregate supply function with the 45°-line Patinkin defines as full employment output (η). He suggests measuring involuntary unemployment by the difference between this full employment output and the actual level of output that results from the intersection of the expenditure function with the 45°-line (Y_0).[6]

In a situation where $\eta > Y_0$, prices would start falling according to Patinkin. Only if price reductions have absolutely no effect on spenders, involuntary unemployment would persist amidst a deflationary spiral (Patinkin 1949, 372). However, thanks to the real-balance effect and the depressing impact of excess supply on interest rates, the expenditure function moves upward, ideally so much that it intersects the 45°-line at

[4] Already in his PhD thesis (submitted in 1947), Patinkin adopted Samuelson's theory of income determination. "This conception probably derived from Lange's teaching and a careful reading of Samuelson's 1941 paper 'The Stability of Equilibrium'" (Rubin 2012, 244).

[5] "It can be shown that the assumptions made in aggregating the supply function imply that the real return to productive services is constant; that is, the price of finished goods is always proportionate to the price of productive services. … (S)ince the real return is constant, suppliers might desire to provide the same amount of goods regardless of the level of income. In that case the aggregate supply function would be a horizontal line- …" (Patinkin 1949, 366). Patinkin abandoned this argument in the reprint of his article (Patinkin 1981) because it contradicts the law of diminishing returns. See also Boianovsky (2002, 230–232)

[6] In modern terms we would call Patinkin's suggested measure of involuntary unemployment the "output gap".

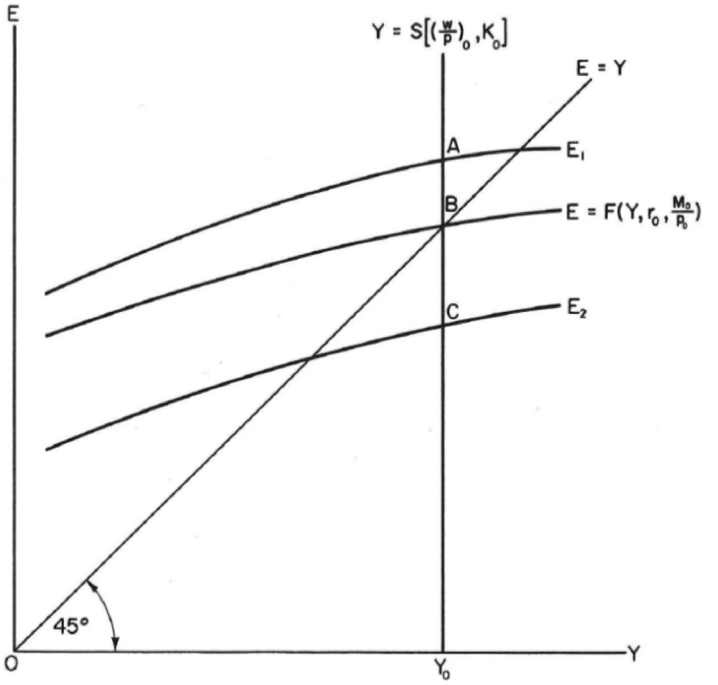

Figure 2. Aggregate supply curve in the Keynesian cross II (source: Patinkin [1956] 1991, Figure IX-3).

the full employment output level (Patinkin 1949, 368, reiterating Patinkin 1948). However, it may also be that the upward push of the expenditure function peters out before full employment is reached. In this case, some involuntary unemployment persists despite falling prices (Patinkin 1949, 372–373).

Also Patinkin's magnum opus *Money, Interest, and Prices* (Patinkin [1956] 1991) contains some clues to aggregate supply. Here, Patinkin drops the idea of a positively sloped (or horizontal) "aggregate desired-supply function" from which he had distanced himself in an unpublished note one year earlier already (Boianovsky 2002, 236). He replaces it with a "familiar aggregate supply function" which had already been present in section 14 of Patinkin (1949) and which results from the production function. For a given real wage, that determines labour input, and a given capital stock a certain output level results from the production function. Therefore, the aggregate supply curve is vertical in the Keynesian cross diagram (Figure 2). "It follows that, for any given real wage rate, the aggregate commodity supply function must appear … as a vertical line drawn at the level of gross national product … for that specified wage rate. As long as this rate remains unchanged, so, too, must this vertical line" (Patinkin [1956] 1991, 211). He goes on stating that a rise in the real wage will shift the aggregate supply function to the left and *vice versa*. As in Patinkin (1949), a drop in demand can result in involuntary unemployment if producers are unwilling to permit their inventories to build up. However, this would lead to a deflation of money wages and prices that restores full employment through the interest-rate and real-balance channels. "Keynesian economics", Patinkin concludes, "overlooks the direct influence of the real-

balance effect on this demand. Similarly, it overlooks the supply side of the commodity market which, by its excess over the demand, generates this effect" (Patinkin [1956] 1991, 325). Interestingly, unlike in his 1949 article, Patinkin in *Money, Interest, and Prices* does not mention the possibility that the upward push of the expenditure function might peter out before full employment (point *B* in Figure 2) is reached. On the contrary, Patinkin [1956] 1991, 324–328) explains that the dynamic process caused by falling wages and prices "cannot stop" (Patinkin [1956] 1991, 325) at a position below full employment.[7]

3. "Keynes' Monetary Thought"

Patinkin (1976) marks a reorientation in his preoccupation with aggregate supply and demand analysis. Although the most common view that, with this book, he turned away from being a mathematical economic theorist towards becoming a historian of economic thought is an oversimplification (Backhouse 2002, 198), the publication of volumes XIII and XIV of the *Collected Writings of John Maynard Keynes* (in 1973) prompted him to refocus from "Keynesian economics" to "the economics of Keynes" (Rivot 2016, 1002). "Patinkin set himself to the task of reviewing the progress of Keynes' ideas towards the principle of effective demand, an endeavor that resulted in a monograph titled *Keynes' Monetary Thought. A Study of its Development*" (Arthmar and Brady 2009, 130).[8]

Patinkin describes the relationship between his new approach and the work discussed above with great candour: "(T)he contention of my 1949 article that there is no supply curve in the *General Theory* was based on a complete an inexcusable failure at the time to understand Keynes' notion of aggregate supply price" (Patinkin 1976, 84, fn. 3).[9] In KMT, Patinkin displays a deeper understanding of the aggregate supply curve. However, for him, "some basic logical difficulties remain" (Patinkin 1976, 84).

Although readers of the present article are probably familiar with the passages from the *General Theory* where Keynes states his aggregate-demand-aggregate-supply (*D/Z*) model, these must nonetheless be reproduced here. Keynes writes:

> Let Z be the aggregate supply price of the output from employing N men, the relationship between Z and N being written $Z = \phi(N)$, which can be called the *aggregate supply function*. Similarly, let D be the proceeds which entrepreneurs expect to receive from the employment of N men, the relationship between D and N being written $D = f(N)$, which can be called the *aggregate demand function*.

[7] See also Boianovsky (2006) on the topics covered in this paragraph.

[8] From what he quotes in KMT we can infer that three types of items published in volumes XIII and XIV of the *Collected Writings of John Maynard Keynes* were instrumental for Patinkin in reconsidering his ideas towards the principle of effective demand: (i) the correspondence between Keynes and his colleagues Roy Harrod, Ralph Hawtrey, Richard Kahn, Nicholas Kaldor, Bertil Ohlin, Dennis Robertson, and Joan Robinson, (ii) lectures contributed by Keynes in the early 1930s, and (iii) the drafts of the *General Theory*.

[9] Further down, Patinkin acknowledges that Keynes' upward-sloping Z curve was similar to his own right- or leftward-shifting vertical aggregate supply function in Patinkin ([1956] 1991). "Accordingly, my criticism then of the *General Theory* on the grounds that it did not provide for an analysis of supply was not well taken" (Patinkin 1976, 91, fn. 12).

> Now if for a given value of N the expected proceeds are greater than the aggregate supply price, i.e., if D is greater than Z, there will be an incentive to entrepreneurs to increase employment beyond N and, if necessary, to raise costs by competing with one another for the factors of production, up to the value of N for which Z has become equal to D. Thus the volume of employment is given by the point of intersection between the aggregate demand function and the aggregate supply function; for it is at this point that the entrepreneurs' expectation of profits will be maximised. The value of D at the point of the aggregate demand function, where it is intersected by the aggregate supply function, will be called *the effective demand* (Keynes [1936] 1973, 25, italics in original).

This quote spells out some fundamental differences between Keynes' aggregate-demand-aggregate-supply (D/Z) model from Chapter 3 of the *General Theory* and the Keynesian cross as alternative ways of representing Keynes' theory. First, the two curves as defined by Keynes are in nominal terms while those of the Keynesian cross are in real terms. Furthermore, Keynes speaks of expectations and profit-maximising. Those elements are absent from the Keynesian cross. Most importantly, Keynes' aggregate supply function Z is not the 45°-line of the Keynesian cross. Z depends on employment, the 45°-line does not; Z has a distinguishable price- and quantity-component, the 45°-line has not. In short: unlike Z, the 45°-line is no autonomous supply function, it is a "helping line" (Samuelson 1948, 257). It is just there to find out which level of income is consistent with the aggregate demand it supports, given the assumptions made about the aggregate demand schedule.

As will be seen, the "evolution of Patinkin's interpretation of Keynes' principle of effective demand" consisted to a large extent in Patinkin adopting changing views as to the meaning and especially the slope of the aggregate supply function Z. With respect to the latter, another passage from a footnote in the *General Theory* is crucial:

> For example, let us take $Z_w = \phi(N)$, or alternatively $Z = W \cdot \phi(N)$ as the aggregate supply function (where W is the wage-unit and $W \cdot Z_w = Z$). Then, since the proceeds of the marginal product is equal to the marginal factor-cost at every point on the aggregate supply curve, we have
>
> $$\Delta N = \Delta A_w - \Delta U_w = \Delta Z_w = \Delta\phi(N),$$
>
> that is to say $\phi'(N) = 1$; provided that factor cost bears a constant ratio to wage cost, and that the aggregate supply function for each firm (the number of which is assumed to be constant) is independent of the number of men employed in other industries, so that the terms of the above equation, which hold good for each individual entrepreneur, can be summed for the entrepreneurs as a whole. This means that, if wages are constant and other factor costs are a constant proportion of the wages-bill, the aggregate supply function is linear with a slope given by the reciprocal of the money-wage (Keynes [1936] 1973, 55–56, fn. 2).

Key for Patinkin's understanding of the aggregate supply function is Keynes' statement that $\phi'(N) = 1$. If $\phi'(N) = \Delta Z_w/\Delta N = \Delta Z/(W\Delta N) = 1$, it follows that $\Delta Z/\Delta N = W$ and that $Z = WN$. This means that Z is a cost function in which N represents all variable-factor inputs. "(T)he aggregate supply price equals total variable costs" (Patinkin 1976, 87, fn. 7).

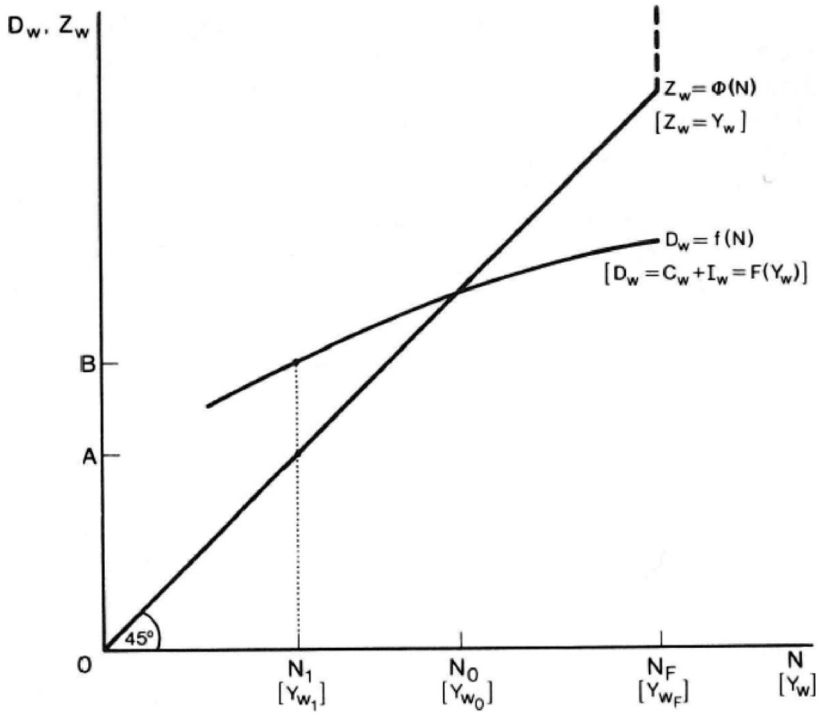

Figure 3. The *D/Z* diagram I (source: Patinkin 1976, Figure 9.1).

Combining Z with the aggregate demand function D yields Figure 3.

The Z_w curve has a slope of 1; hence it is the 45°-line ($\Delta Z_w = \Delta N$). At employment level N_1, "(t)he corresponding ordinate OA represents *actual* costs of production of that output. In contrast, the ordinate OB represents not the *actual* proceeds but the *expected* ones. Why, then … does Keynes treat the difference between OA and OB as if it represented actual, realized profits that motivate the entrepreneur to expand output? And even more puzzling, why does Keynes contend that profits are at a maximum at the point of intersection of his demand and supply curves, where profits as measured by the foregoing difference are zero?" (Patinkin 1976, 90, italics in original).

So Patinkin's interpretation of the *D/Z* model is in conflict with Keynes' claim that producing an output corresponding to the effective demand maximises expected profits. This is not the only interpretative problem, however. Also, in the last sentence of the "footnote", Keynes suggests that the slope of the aggregate supply curve equals "the reciprocal of the money-wage". Surely, the slope cannot both be equal to 1 and to the reciprocal of the money-wage, unless the latter also equals 1. Or does Keynes mean the slope of the Z curve as opposed to the Z_w curve in the last sentence? This is what Patinkin (1976, 88) believes to be true. However, as was shown above, if the slope of the Z_w curve equals 1, then the slope of the Z curve equals the money wage-unit (W), not its inverse. Therefore, Patinkin claims that the word "reciprocal" "should not appear here, or else it refers to the measurement of the slope with respect to the vertical axis" (Patinkin 1976, 88, fn. 8).

If the system converges to N_0 in Figure 3, then proceeds will just cover costs. This, in Patinkin's (1976, 87–88) reading, Keynes means by "proceeds which will just make it worth the while of the entrepreneurs to give that employment" (Keynes [1936] 1973, 24). However, this ignores that Keynes adopted Marshallian microfoundations, which assume that entrepreneurs maximise profits not that they aim at covering costs.[10] Also, Keynes explicitly states in the passage quoted above that "the entrepreneurs' expectation of profits will be maximised" at the point of effective demand. Patinkin therefore suggests "that these words should simply be deleted from the *General Theory*" (Patinkin 1976, 93).

As was mentioned in the Introduction, two journal articles complement Patinkin's analysis in KMT. The "correction" Patinkin (1977) found necessary to make refers to a minor issue. In Figure 3 (reproduced from Patinkin 1976) he placed income in wage-units in square brackets below the employment levels. Patinkin soon realised that with respect to real income, the 45°-line no longer measures the minimum proceeds (costs) on which firms will insist since its unitary slope now represents the marginal return from this output. By definition, if output rises by one unit, its total value rises by one unit. Hence, with respect to Y_w the 45°-line represents the total market value of output (variable costs plus profits in wage units) and not the aggregate supply price as defined by Patinkin in KMT (total variable costs). Since he did not attribute this part of his argument to Keynes, this error does not bear on Patinkin's interpretation of the theory of effective demand in the *General Theory*.[11]

Patinkin (1978) is a defence of his interpretation of the Z function in KMT against a (mostly critical) review by Roberts (1978). Patinkin reiterates that, based on the "footnote" – or rather the upper part of it – the Z function (in wage-units) must be interpreted as representing total factor costs (Patinkin 1978, 585). But then he makes a concession that paves the way for his reinterpretation of Keynes' theory of effective demand vis-à-vis KMT that will be discussed in the next section. He writes that "Keynes *incorrectly* defines a supply curve as identical with the total-variable-cost curve" (Patinkin 1978, 586, italics added). Vaguely, he allows the aggregate supply curve to be derived from profit-maximisation. It then no longer coincides with the 45°-line. The proposition "that *every* point on the aggregate supply curve is a point of maximum profits" he declares "valid" (Patinkin 1978, 590, italics in original). This is all quite surprising against the background of KMT. Patinkin refrains from explaining these statements, but refers (in fn. 24) to his forthcoming article (Patinkin 1979a) for "a more systematic and detailed discussion of these points". He summarises his new view as follows: "There can be little doubt that aggregate supply price as specified in

[10] Asimakopulos (1982, 18) insists that "Keynes accepted implicitly Marshall's microeconomics, and … they provided the foundations for his aggregate supply function". He also notes that Keynes' choice of words in the *General Theory* just "echoes Marshall's definition of the supply price for a particular commodity, 'the normal supply price of any amount of that commodity … is the expectation of which will just suffice to maintain the existing aggregate amount of production'" (Asimakopulos 1982, 23 citing Marshall 1920, 342–343). The proceeds which will just make it worth the while of the entrepreneurs to give that employment "include all the elements, other than user costs, to be found in industry supply curves, that is, labour, other factor costs and profits" (Asimakopulos (1982, 25). Asimakopulos (1982, 32) also criticizes that Patinkin's Z curve in Figure 3 becomes vertical at full employment N_F because Z is derived from individual firm's curves which are drawn "without reference to possible constraints on output due to shortages of labour or other variable inputs". So if the individual curves do not have vertical parts, their aggregate counterpart should neither.

[11] In the German translation of KMT (Patinkin 1979b), the Y_w terms in square brackets are gone.

this footnote is equal to total variable costs. And though this specification is analytically incorrect …, no clear evidence has been supplied that Keynes specified aggregate supply price in any other way at other points in the *General Theory*" (Patinkin 1978, 591).

The next task Patinkin set himself to was to specify the aggregate supply function correctly. The next section will show that Patinkin dropped his claim that the aggregate supply curve is the 45°-line. His new claim is that Keynes did not properly understand his own Z function.[12]

4. "A Study of Keynes' Theory of Effective Demand"

In Patinkin (1979a), he argues more formally than in his earlier contributions on aggregate supply. He opens up his argument stating: "let us first derive an aggregate supply function from the principle of profit-maximization. Accordingly, let the production function be

$$Y = \psi(N)" \tag{1}$$

(Patinkin 1979a, 158).

As stated in the Abstract, my aim in this paper is not only to trace the evolution of Patinkin's interpretation of the theory of effective demand, but also to comment on it. Patinkin's attribution of a macroeconomic production function (from which to derive the aggregate supply function) to Keynes might be questioned. Keynes, in Chapter 4 of the *General Theory*, expressed concern over the concept of aggregate real output – Y in formula (1) – because real output was not homogeneous. He declared that he wanted to use only two units for measuring macroeconomic aggregates, namely money and labour. Therefore, authors such as Hayes (2007) and Ambrosi (2011) have denied the admissibility of the aggregate production function for a reconstruction of Keynes' theory.[13] However, Keynes occasionally used the notion of aggregate real output himself – on p. 209 of the *General Theory*, for instance, he calls it O –; and it is a very useful and established concept in macroeconomics.[14] So I would go along with Patinkin in taking the aggregate production function with labour as the only variable factor of production as starting point.[15]

Next, Patinkin writes down the first-order condition as

$$w/p = \psi'(N) \tag{2}$$

and calls it the "labor-demand function" (Patinkin 1979a, 158).

[12] "(M)y basic criticism of Keynes' presentation of his aggregate supply function is that it is a confused one" (Patinkin 1978, 588).

[13] An anonymous reviewer for this journal seems to agree claiming that the use of a production function implies that the variables appearing in Figure 2 above and in Figure 4 below are defined in terms of a single good produced that can be consumed or invested. According to the reviewer, in such an economy, the aggregate demand function would not be independent of the aggregate supply function or the 45°-line in the Keynesian cross; and Say's Law would apply.

[14] See Hartwig and Brady (2008) for a critique of Hayes (2007). Froyen (1976) argued that Keynes finally opted for a single measure of output in the *General Theory*.

[15] In fact, I did exactly this when I reconstructed the Z function in Hartwig (2000, 174–179). At that time, however, I was unaware of Patinkin (1979a) and used Chick (1983) as my main point of reference.

Arguably, if Keynes had accepted the neo-classical labour-demand function, it would hardly have been necessary for him to devise a new ("General") theory of employment. I fully agree with Asimakopulos (1982, 30) that "(t)here is no separate independent demand curve for labour in Keynes' model. … (The) inverse relationship [between employment and the real wage] should not be confused with a labour-demand curve … There is no labour market in Keynes' model in which labour demand and labour supply curves interact to determine the equilibrium level of employment and the real-wage rate".[16] Nonetheless, the first-order condition is very important as it picks up the idea of profit-maximisation. Following Chick (1983), my approach in earlier publications has been to solve Equation (2) for the price level, to call this the "supply price level", then to multiply this supply price level with the output level from Equation (1) to get $Z = w \cdot \frac{\psi(N)}{\psi'(N)}$ or $Z_w = \frac{\psi(N)}{\psi'(N)}$.[17] Patinkin argues slightly differently. He first expresses the production function (1) "in money terms" and deflates it by the wage-unit (w) to get

$$Y_w = \frac{p \cdot \psi(N)}{w}. \tag{3}$$

"Upon substitution from profit-maximizing condition (2), this becomes the aggregate supply function

$$Z_w = \frac{\psi(N)}{\psi'(N)}," \tag{4}$$

(Patinkin 1979a, 159–160).

Applying the quotient rule and assuming diminishing marginal productivity ($\psi''(N) < 0$) it is easy to show that the slope of the Z_w curve is greater than unity.[18] So Z_w lies above the 45°-line, which represents variable costs. This is Patinkin's new insight vis-à-vis Patinkin (1976).

From the perspective of the Chick-Hartwig interpretation – if I may call it that way – as detailed in Chick (1983), Hartwig (2000) and elsewhere, Patinkin's approach to substitute Equation (2) into Equation (3) is not quite satisfactory as it ignores that there are two price levels involved in the D/Z model, not just one.[19] The supply price is not the market price level an entrepreneur *expects*, but the proceeds he must have for the last unit of output at each level of employment to satisfy the profit maximising condition. This unit supply price will grow with employment under conditions of decreasing

[16] The English translation of Hartwig (2000) is *Keynes vs. Pigou. Reconstruction of an Employment Theory Beyond the Market Paradigm*. See also Davidson (1983).

[17] Note that for employment to rise by one unit the "supply price level" has to increase under conditions of diminishing returns. Hence, the real wage must fall and income must be transferred from the previously employed workers to the entrepreneurs. This principle enshrined in Keynes' aggregate supply function is in keeping with Pigou's ([1933] 1968) *Theory of Unemployment* (see also Arthmar and Brady 2009, 139–140).

[18] $\frac{dZ_w}{dN} = 1 - \frac{\psi(N)\psi''(N)}{[\psi'(N)]^2} > 1$.

[19] Interestingly, Patinkin had seen that in KMT, where he wrote: "In graphical terms … Keynes seems to have treated the difference between OA and OB in Figure 9.1 [Figure 3 in this paper] as if it represented the difference between two different per-unit prices that actually existed in the economy at a given level of aggregate output" (Patinkin 1976, 92).

marginal returns to labour. P^s, the price level implicit in Z, is in a way purely hypothetical. *If*, for a certain N_1, the entrepreneurs expected the price level $P^s = w \cdot \frac{dN_1}{dY}$ to rule in the market they would employ N_1 men because they knew that profits would thereby be maximised. But which price level do they *really* expect? This question is not answered by the aggregate supply function at all but by the aggregate demand function. The price level implicit in D, which I call the "demand price level" P^d, is the price level the entrepreneurs really expect to rule in the market and which they must "take" under free competition that Keynes assumes. Being profit-maximizers, the entrepreneurs must be on the aggregate supply curve – they cannot be off that curve. All points on Z are profit-maximising. The aggregate demand curve D picks the "right" profit-maximising level of employment where it intersects Z at the point of "effective demand".

In my most recent contribution to the D/Z controversy (Hartwig 2017), I use specific functional forms for the D and Z functions and run numerical simulations which allow study of the comparative statics of the model in the face of various "shocks". I use the production function $Y = N^{\alpha}$ with $\alpha = 0{,}7$ as starting point. As chance would have it, Patinkin (1979a, 161) uses almost the same function $Y = AN^{\alpha}$ (with $0 < \alpha < 1$) for illustrative purposes.[20] This production function gives rise to the following Z_w function (depicted in Figure 4)

$$Z_w = \frac{AN^{\alpha}}{\alpha AN^{\alpha-1}} = \frac{1}{\alpha}N \tag{5}$$

The slope of Z_w, $(1/\alpha)$, is greater than unity, which is the slope of the total variable cost (TVC) curve. "By construction, every point on this aggregate supply curve is a point of maximum profits – for the real-wage rate to which it corresponds" (Patinkin 1979a, 161).[21] For the aggregate demand curve $D_w = {}^1f(N)$ in Figure 4, $0A = N_1T$ are total proceeds, N_1U are wages and UT are profits (all in wage-units). "In accordance with the property of a Cobb-Douglas function, the share of wages in the total value of output … is constant and equal to

$$\alpha = N_1U/N_1T = N_2S/N_2R"$$

(Patinkin 1979a, 162).

This means that the slope of the Z curve is equal to the inverse of the wage share (α) or, in other words, equal to the inverse of the output elasticity, which is equal to the wage share for a Cobb–Douglas production function. Although this insight pre-dates Patinkin (1979a),[22] it was Ambrosi (2011) who seems to be the first to have suggested that it can be used to solve one of the puzzles posed by the "footnote" on pp. 55–56 of

[20] I did not quote Patinkin (1979a) in Hartwig (2017) because I discovered his article only recently when collecting literature for the present paper. Patinkin (1979a) has not received much attention. The article has received only 12 citations in Scopus (as of November 2021); and no reviewer or editor has ever pointed my attention to it. See Patinkin (1982b) on "multiple discoveries" in science.

[21] I would replace "real-wage rate" with "marginal product of labour" as Patinkin's wording reflects his substitution of the "labour-demand function" into the production function, which was criticized above. However, since we assume that the supply price level moves to equate the real wage with marginal product, Patinkin's formulation is correct.

[22] The earliest statement I found is by Marty (1961) who wrote: "The tangent of the angle made by drawing a line from the supply function to the origin is the reciprocal of the relative share of labour. If the relative share

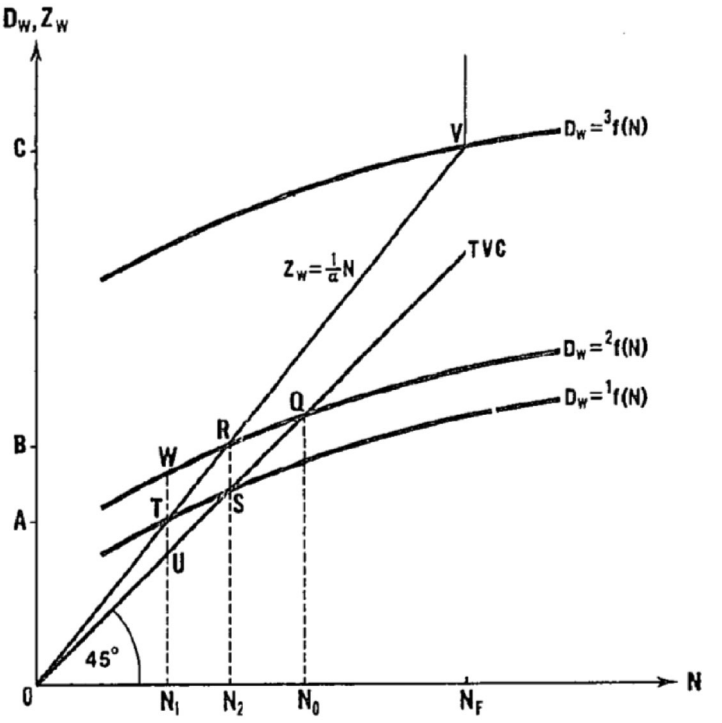

Figure 4. The *D/Z* diagram II (source: Patinkin 1979a, Figure 2).

the *General Theory*. In the last sentence of the footnote, Keynes claims that the slope of the aggregate supply function equals the inverse of the money wage. Ambrosi agrees that this is wrong, but he claims that the statement could be corrected simply by inserting the word "share" at the very end of the footnote. In other words, Ambrosi thinks that Keynes "really meant" that the slope of the aggregate supply function is equal to the reciprocal of the money wage *share*. Although it cannot be proved that Keynes' pen "slipped" here,[23] this is an attractive explanation. Hartwig (2017, 362–363) offers a mathematical proof that the slope of Z_w is equal to the inverse of the output elasticity.[24]

In section III of his article, Patinkin (1979a) collects passages from the *General Theory* which accord with his new interpretation of the theory of effective demand. However, in section IV, he rejects the idea that this was Keynes' own interpretation. His argument is twofold. First, there is the "footnote" where Keynes states that the slope of the aggregate supply function in wage-units equals 1. This means he must have thought that Z_w is the 45°-line. And second, neither in Keynes' *Treatise on Money* nor in the drafts of the

is constant as we move along the supply function we trace a linear function. Linearity of the supply function implies constancy of the relative share" (Marty 1961, 561–562). Patinkin's (1979a, 159, fn. 5) statement that Marty has not analyzed the "slope and other properties" of Z is thus incorrect. See also Davidson and Smolensky (1964, 125, 134–135), Asimakopulos (1982, 26) and Arthmar and Brady (2009, 140) on $(1/\alpha)$ being the slope of Z_w.

[23] See Heller (2009).

[24] This argues against Patinkin's (1976, 88) conjecture that the last sentence of the "footnote" was about the slope of Z instead of Z_w.

General Theory can references to profit-maximisation and marginal analysis be found. So ignoring the fact that the passage from p. 25 of the *General Theory* quoted above is perfectly in line with his new interpretation, Patinkin concludes that, *for Keynes*, D represents expected proceeds and Z (erroneously) total variable costs.

The last critique Patinkin offers is that "even if Keynes had correctly derived his aggregate supply curve from profit maximisation (thus yielding, say, $Z_w = (1/\alpha)N$ in Figure 2 [Figure 4 in this paper]), its intersection with $D_w = {}^2f(N)$ at R is not a point of maximum profits" (Patinkin 1979a, 171). To prove this, Patinkin first redefines the aggregate demand curve $D_w = {}^2f(N)$ in Figure 4 as representing *actual* proceeds (Patinkin 1979a, 172–173), even though he admits, that for Keynes it represented *expected* proceeds.

Given that Patinkin's aim for his article was "to determine to the best of my ability what he [Keynes] did say" (Patinkin 1979a, 155), this departure from his source seems odd. I can only speculate about his reasons. Maybe these had to do with the problem of how to arrive at aggregate results starting out from the expectation formation of individual entrepreneurs. On pp. 24-25 of the *General Theory*, Keynes describes D and Z both as functions of N. But while N in the entrepreneurs' "micro" Z functions stands for the employment in the entrepreneurs' own firms, and the individual Z functions can be aggregated straightforwardly, the N in the "micro" D functions cannot stand for the employment in the entrepreneurs' own firms because demand expectations of individual entrepreneurs do not depend on the employment they give themselves. Therefore, Asimakopulos (1982) rejects the expectational D function, and he quotes Patinkin (1979a, 172–174) as a point of reference (see Asimakopulos 1982, 21, fn. 10).[25] Patinkin's argument seems to be somewhat different, though. He accepts that individual entrepreneurs are (demand-) price takers because they cannot observe the aggregate demand function. For them, "Keynes' description of the point of intersection of the aggregate demand and supply curves as one of maximum profits … is correct" (Patinkin 1979a, 173). For the "firms as a whole, operating as one unit" (Patinkin 1979a, 173), however, things are different. The "unit" can perceive the aggregate demand curve – the actual one that is, not Keynes' expectational one – and can move freely on it, according to Patinkin. If the "macro" D curve is $D_w = {}^2f(N)$ in Figure 4 – as Patinkin has postulated (see above) – then the difference between actual proceeds on $D_w = {}^2f(N)$ and total variable costs on the 45°-line – hence profit – is higher than at the point of effective demand R when employment falls below N_2.[26]

Patinkin comes to this conclusion because he treats the aggregate of firms like a monopolist who can move freely on the D curve. I have doubts that the aggregation problem should be solved this way. In my interpretation (see fn. 25 above), the D curve is exogenous for the entrepreneurs, as is the price component

[25] I think Asimakopulos' rejection of the expectational D function is unwarranted. Entrepreneurs do form expectations on *overall* employment – or the state of the business cycle, respectively – when they decide how many workers to employ in their own firm. So "macro" employment and "micro" employment are interrelated. See Hartwig (2004, 81–82) for a defence of the expectational D function.

[26] See also Boianovsky (2002, 248–249) on the topics covered in this paragraph and the next.

inherent in it. Entrepreneurs are (demand-) price takers. They cannot move freely on D. On the other hand, being the suppliers, entrepreneurs cannot be off the aggregate supply curve. To repeat from above, the aggregate demand curve D picks the "right" profit-maximising level of employment where it intersects Z at the point of "effective demand".[27]

Howsoever, Patinkin softens his criticism of D in his concluding examination of the theory of effective demand. Patinkin (1982a) is basically a reprint of Patinkin (1979a), albeit with some rearrangements and amendments. One such rearrangement concerns the two criticisms from section IV of Patinkin (1979a). Patinkin (1982a) now discusses the "inconsistency" concerning D – that profits are not maximised at the point of intersection with Z – first and indicates that it can be removed when D is reinterpreted as representing actual instead of expected expenditure (Patinkin 1982a, 143–144). It will be remembered that this redefinition was the first step in his proof of the "inconsistency" concerning D in Patinkin (1979a, 172–173). The "inconsistency" arose because Patinkin assumed that the entrepreneurs "as a unit" could perceive and move on $D_w = {}^2f(N)$ in Figure 4. If, however, "the aggregate demand curve is *not* perceived by entrepreneurs" (Patinkin 1982a, 143, italics in original) so that $D_w = {}^2f(N)$ represents the total consumption and investment expenditures, and if $D_w = {}^1f(N)$ represents aggregate demand as expected by the entrepreneurs, then at employment level N_1 actual demand is higher than expected demand. So the entrepreneurs have an incentive to expand output. This triggers a quantity reaction – just like an upward shift of the demand curve in the Keynesian cross would – which continues until the economy reaches an equilibrium at point R in Figure 4, "where, for the representative competitive firm, market price equals marginal cost. Thus, from the viewpoint of such a firm, R is a point of maximum profits in the sense that the firm has no incentive to depart from it" (Patinkin 1982a).

So with respect to D, Patinkin eases his criticism vis-à-vis Patinkin (1979a) somewhat. He maintains, however, that "Keynes did not distinguish properly between propositions which are valid for the representative firm operating under conditions of perfect competition and those valid for firms as a whole, operating as one unit" (Patinkin 1982a, 144).

The "second inconsistency" – the "footnote's" statement that $\phi'(N) = 1$ – "is a much more serious one" (Patinkin 1982a, 144). Patinkin offers a new insight, how this statement might be explained. He argues that Keynes simply confounds the aggregate supply function $\phi(N)$ with the production function $\psi(N)$ in the "footnote". For it follows from the first-order condition (Equation 2 above) that $\frac{p}{w}\psi'(N) = 1$. That is, the slope of the production function (Equation 1) "in money terms" and deflated by the wage-unit (w) equals 1. "Was Keynes' implicit, erroneous identification of the production and supply functions transitory or permanent?", Patinkin asks. "Was it a chance error of this footnote or a systematic component of his thinking? The evidence is not clear, but I think the latter alternative is closer to the truth" (Patinkin 1982a, 145). Which leads Patinkin to his final verdict on Keynes' theory of effective demand: "And

[27] Victoria Chick was right to point out that "(e)ffective demand is an unfortunate term, for it really refers to the output that will be *supplied*; in general there is no assurance that it will also be demanded" (Chick 1983, 65).

this is my main point: that the obscurity with which the aggregate supply curve is presented in the *General Theory* is a sign not of profundity, but of obscurity" (Patinkin 1982a, 150).

5. Conclusion

The evolution of Patinkin's interpretation of Keynes' theory of effective demand made some turns. Patinkin started out supplementing the then-dominant way of representing Keynesian theory – the Keynesian cross – with an aggregate supply function. His initial version of that function, in Patinkin (1949), was nearly horizontal; later on, in *Money, Interest, and Prices*, Patinkin opted for a vertical version of the aggregate supply function. Failure to include aggregate supply in the Keynesian cross let "Keynesian economics", in Patinkin's view, to neglect that an excess of supply over demand would generate a deflation of money wages and prices that would restore full employment through the interest-rate and real-balance channels.

After the publication of volumes XIII and XIV of the *Collected Writings of John Maynard Keynes* (in 1973) Patinkin refocused from "Keynesian economics" to "the economics of Keynes". The drafts of Keynes' *General Theory* as well as the correspondence between Keynes and his colleagues published in these volumes were instrumental for Patinkin in recognising the importance of Chapter 3 of the published version of Keynes' book titled "The Principle of Effective Demand". However, Patinkin struggled for several more years with identifying the true meaning of this "most important and … most obscure chapter in the *General Theory*" (Patinkin 1979a, 155). His main interpretative problem was with Keynes' aggregate supply function Z. Chiefly based on a footnote from pp. 55–56 of the *General Theory*, in which Keynes states that the slope of the aggregate supply function in wage-units (Z_w) equals 1, Patinkin identified the latter as the total-variable-cost curve in his book *Keynes' Monetary Thought* (Patinkin 1976). He was well aware, however, that several passages of the *General Theory*, for instance Keynes' insistence that profits are maximised at the point of effective demand – which is the point of intersection of the aggregate supply function Z and the aggregate demand function D – were at odds with this interpretation. Therefore, in 1979, he revisited his interpretation, integrating the first-order condition in Z. The slope of the aggregate supply function in wage-units (Z_w) in the Z_w/N-space is consequently greater than 1, and the distance between Z_w and the 45°-line measures profits (in wage-units).

This interpretation, Patinkin was convinced, was the correct version of the aggregate supply function. This implies that the version in Patinkin (1976) is wrong.[28] I agree with Patinkin on this point. Patinkin attributes the wrong version – the KMT version – to Keynes, however.[29] His main evidence: the "footnote" from pp. 55-56 of the *General Theory*. I tend to disagree with Patinkin on this second point because the interpretation of Keynes' principle of effective demand developed in Patinkin (1979a) and reaffirmed

[28] "The chapter is a further development – and in part a correction – of the critique of Keynes' theory of effective demand which appears in chapter 9 of KMT" (Patinkin 1982a, 123).

[29] "The analysis of section 3 above not only demonstrates the invalidity of Keynes' description of the supply curve as having a constant unitary slope, but also suggests that the immediate cause of Keynes' error was his failure to distinguish between the supply function and the production function" (Patinkin 1982a, 145).

in Patinkin (1982a) is perfectly in line with the passage on p. 25 of the *General Theory* quoted above in section III.

There must be something wrong with the "footnote", however, since it makes conflicting claims about the slope of Z_w, namely that the latter equals at the same time 1 and the reciprocal of the money wage. I already mentioned Ambrosi's (2011) proposal to rectify the last sentence of the footnote by inserting the word "share" at the very end and Patinkin's (1982a) suggestion that it should read $\frac{p}{w}\psi'(N) = 1$ instead of $\phi'(N) = 1$ in the upper part of the "footnote". Another suggestion can be made. Keynes [1936] 1973, 283) writes that "if the elasticity of output is unity, no part of the increased effective demand is expected to accrue as profit". In this special case of zero marginal profits, the slope of the aggregate supply function in wage-units (Z_w) equals 1. So the statement $\phi'(N) = 1$ is correct, but only for the special case of an output elasticity – and hence a wage share – of 1.[30]

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[30] See also Marty (1961, 563).

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